

Determination Of Quick Switching System-Ss (Two Single Sampling Plans) Using Poisson Distribution

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Received: 15 Feb 2025 Accepted & Reviewed: 25 Feb 2025, Published : 28 Feb 2025

Abstract

Quick Switching Systems (QSS) are generally employed in making acceptance/rejection decisions during inspection process of any manufactured product. Based on the past results, the number of units and the acceptance criteria may be varied in QSS hence provide additional protection during poor quality time and reduction in inspection cost during good quality time. This article focuses on protection provided by QSS-SS (two single sampling plans) using Poisson distribution during the time of constant and changing quality. QSS-SS are compared with Single Sampling Plan, Double Sampling Plan and Quick Switching System where QSS-S, where QSS-SS was given by Taylor in the year 1992 and showed the efficiency with QSS (Romboski).

Keywords: QSS, QSS-SS, Poisson distribution, Stationary OC curve, Transitive OC curve, ASN, MTBS.

Introduction

QSS was originally proposed by Dodge (1967) and investigated by Romboski (1969) and Govindaraju (1991). Taylor in 1992 redesigned the system and constructed the tables of Quick Switching System and a program to select and evaluate QSS. Later in 1996, Taylor constructed QSS -SS with Binomial Distribution as a baseline distribution and framed the Stationary and Transitive OC curves. This article demonstrates the evaluation and selection of QSS-SS using the Poisson distribution as a baseline distribution for Acceptance Sampling. This method describes the protection provided by the sampling plans during the time of changing quality referred to as Transitive Operating characteristic Curves (OC). The comparison is made between Single, Double and Quick Switching Systems.

The Quick Switching Systems examined in this article hold two sampling plans such as reduced plan and tightened plan with certain rules in the switching process. The Reduced plan, first sampling plan utilized during the time of good quality with smaller sample size reduces the cost of inspection. The Tightened plan, second sampling plan utilized during the problems met. The tightened plan is designed to provide the high level of protection. The switching rules are easy to use and assure that the correct plan is utilized and react quickly to the quality changes.

The Quick Switching System designated as QSS-SS; the last two SS represents two Single Sampling plans. One starts with tightened plan, for each lot inspected, two decisions are made firstly whether to accept or reject the current lot and secondly which two sampling plans to use for the next lot. To accomplish this task, each sampling plans hold a related switch number. Figure 1 demonstrates the work of Switching rules.

The QSS-SS explained in this article vary from QSS proposed by Dodge in that the decision to switch is made separate from the decision to accept/ reject.

Operating Procedure

- Take a random sample of size ' n_t ' at tightened inspection and count the number of defectives ' d_t '.

- If $d_t \leq s_t$ (tightened switch number), where $s_t = a_t$ (acceptance number), switch to the Reduced inspection.
- In Reduced inspection, again take a random sample of size ‘ n_r ’ and count the number of defectives ‘ d_r ’.
- If $d_r \geq s_r$ (reduced switch number), where $s_r = a_r + 1$ (acceptance number), again switch to Tightened plan.

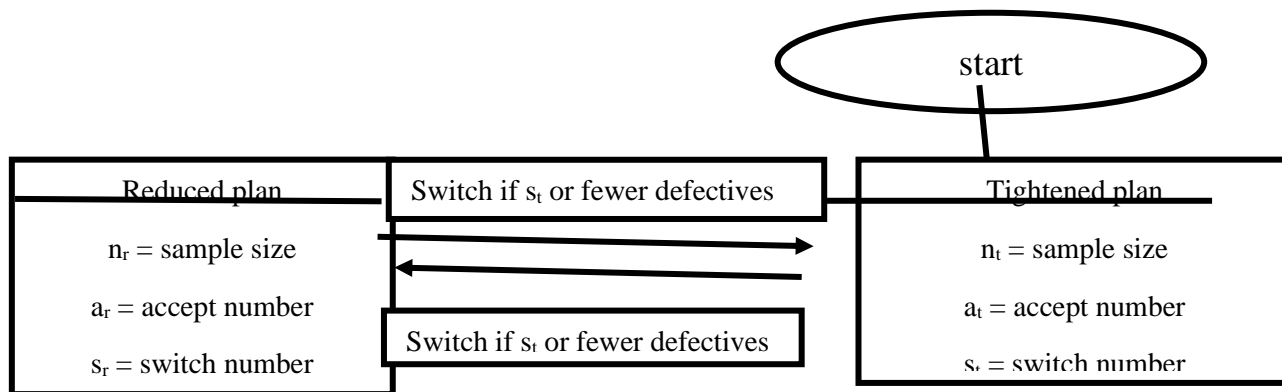


Figure 1: Flow chart for Switching rules – QSS-SS

Stationary OC curve:

Operating characteristics curves shows the protection provided by the sampling plans and assess whether the probability of acceptance depends on the lot or process quality. The lots are inspected independently in single and Double Sampling plans therefore the protection does not depend on the past results. But QSS-SS uses past results to determine how to inspect or to decide whether to accept the current lot. A lot is rejected when the previous lots were rejected and a lot is accepted when the previous lots were accepted. Hence, the OC curve explain the protection under different periods of quality. To fulfil this issue, two types of OC curves are used namely, Stationary OC curve and Transitive OC curve.

The Stationary OC curve is denoted by $OC_s(p)$. this curve provides the protection of QSS under the stationary conditions namely, a long series of lots of the same quality. When the process fraction defective is ‘ p ’,

$OC_r(p)$ - probability of acceptance for the reduced plan

$S_{r \rightarrow t}(p)$ - probability of switching to tightened when in reduced

$OC_t(p)$ - probability of acceptance for the tightened plan

$S_{t \rightarrow r}(p)$ - probability of switching to reduced when in tightened

Under Stationary conditions, the OC curve for QSS is defined by,

$$OC_s(p) = Pr_r(p) OC_r(p) + Pr_t(p) OC_t(p) \tag{1}$$

Where, $Pr_r(p) = \frac{S_{t \rightarrow r}(p)}{S_{t \rightarrow r}(p) + S_{r \rightarrow t}(p)}$ and $Pr_t(p) = 1 - Pr_r(p)$ (2)

$Pr_r(p)$ and $Pr_t(p)$ are the fraction of time spent in reduced and tightened states. When defectives are tallies, QSS-SS plan is utilized and hence,

$$OC_r(p) = P(a_r | n_r, p)$$

$$S_{r \rightarrow t}(p) = 1 - P(s_r - 1 | n_r, p)$$

$$OC_t(p) = P(a_t | n_t, p) \quad (3)$$

$$S_{t \rightarrow r}(p) = P(s_t | n_t, p)$$

$P(x|n, p)$ is the Poisson distribution function representing the probability of x or fewer defectives in a sample of n units from a process average p fraction defective.

Figure 1 describes the stationary/composite OC curve for the QSS-SS $n_r=20$, $a_r=0$, $s_r=1$ and $n_t=60$, $a_t=0$, $s_t=0$, individual OC curves for the reduced and tightened plans and OC curve of SSP with $n=80$ and $a=1$. The Stationary OC curve always lie between the reduced and tightened OC curves since it is the weighted average of these two curves. The Stationary OC curve tend towards the tightened plan OC for high defective rate and towards reduced plan for lesser defective rate. From the figure 2, the stationary OC curve closely approximates the Single Sampling Plan and the probability of acceptance is higher compared with the Quick Switching System. But, QSS-SS provides this protection during the periods of constant quality. When the process quality changes, QSS-SS has increased risk of accepting the bad lots and rejecting the good lots. The Transitive OC curves are also helps to determine the increase in risk during the time of changing quality.

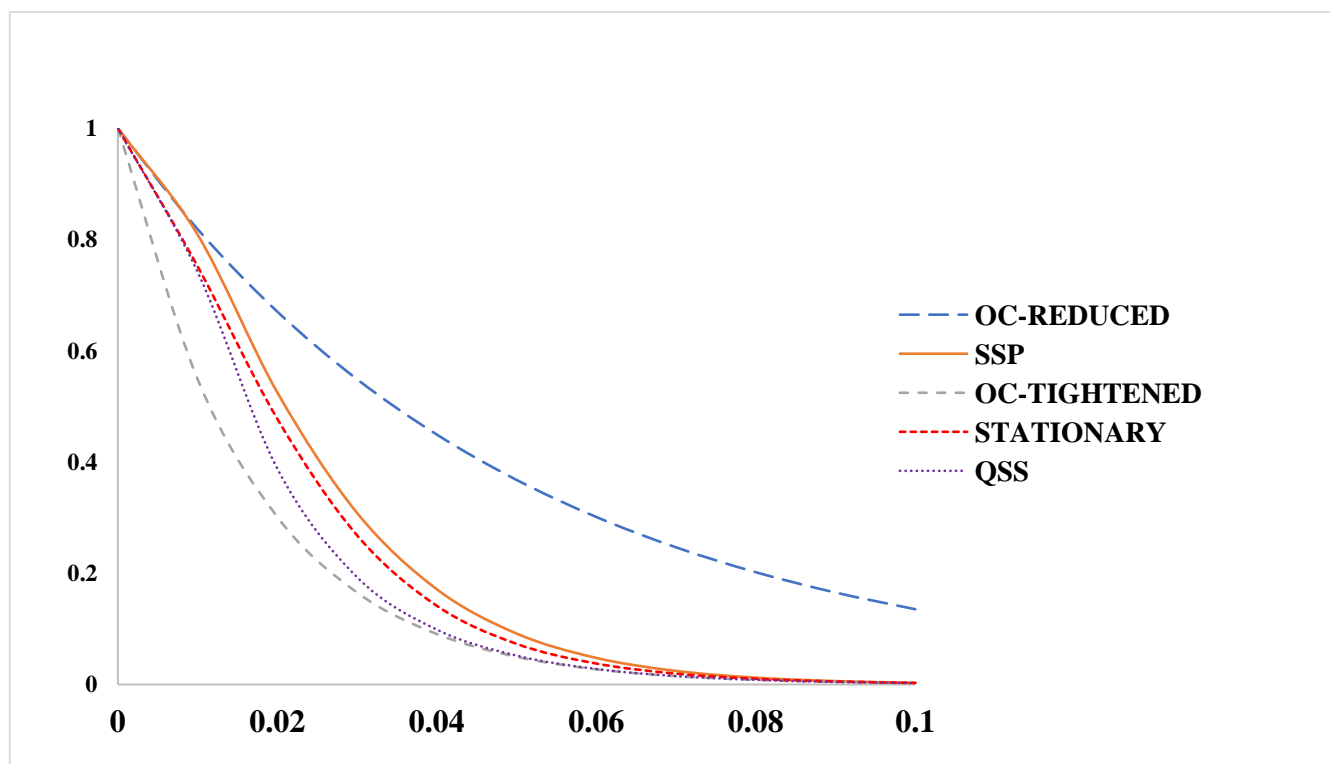


Figure 2: Stationary, Reduced and Tightened OC Curves

Transitive OC curve:

These curves quantify the protection provided during the periods of changing quality. Here, considers the process has been running p_{old} fraction defective, suddenly shifts to p_{new} fraction defective and that we are

inspecting the n^{th} lot following the change. The probability of acceptance for the transitive OC curve is denoted by $OC_T(p_{new} | p_{old}, n)$. If $Pr_r(p_{new} | p_{old}, n)$ is the probability that the reduced plan is used for inspection, then

$$OC_T(p_{new} | p_{old}, n) = Pr_r(p_{new} | p_{old}, n) OC_r(p_{new}) + (1 - Pr_r(p_{new} | p_{old}, n)) OC_t(p_{new}) \quad (4)$$

where, $Pr_r(p_{new} | p_{old}, 1) = Pr_r(p_{old})$ and

$$Pr_r(p_{new} | p_{old}, n+1) = Pr_r(p_{new} | p_{old}, n) (1 - S_{r \rightarrow t}(p_{new})) + (1 - Pr_r(p_{new} | p_{old}, n)) S_{t \rightarrow r}(p_{new}) \quad (5)$$

Figure 2 shows the different transitive OC curves for the periods of changing quality. These curves assume that the previous lots are all zero percent defective. Under these conditions, the probability of accepting the first 6% defective is 0.449 and the probability of accepting the second 6% defective is 0.301. when the process quality remains unchanged, the transitive OC curve converge back to the Stationary OC curve.

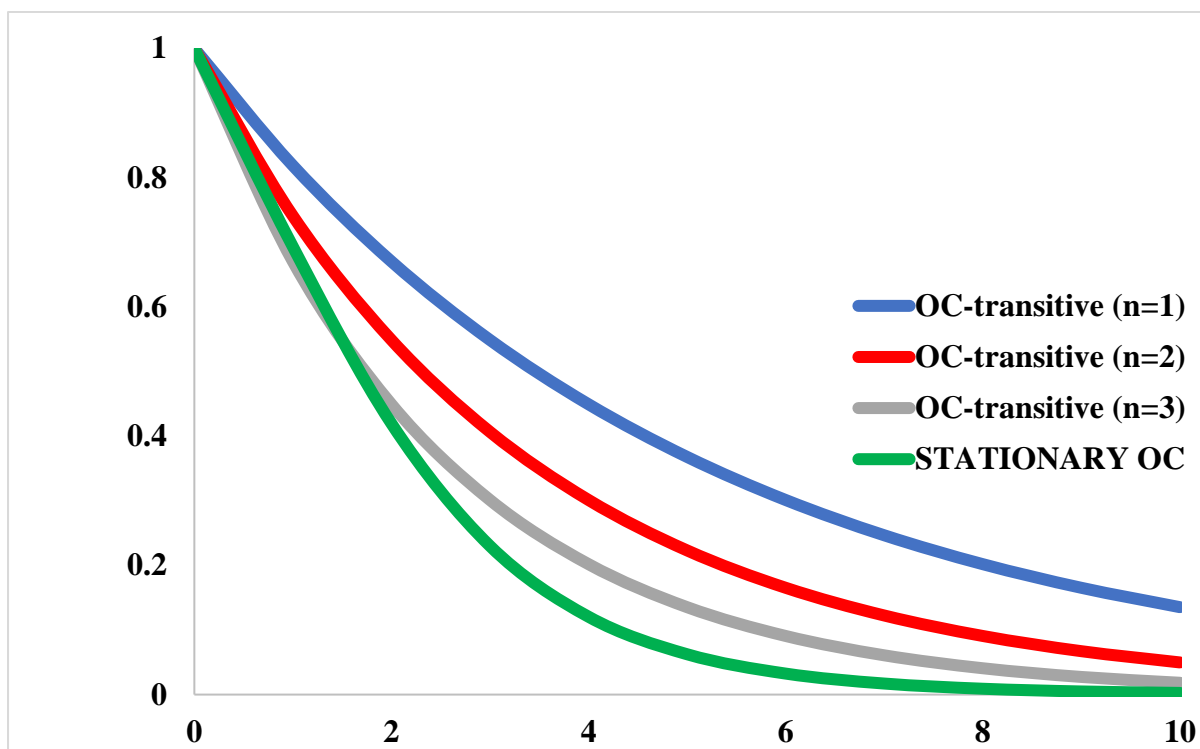


Figure 3: Transitive OC Curves

AQL and LTPD:

The AQL (Acceptance quality Level) represents the quality level routinely produced by the sampling plan. The rejection probability 0.05 at AQL is defined as Producer’s risk or alpha risk. The LTPD (Lot Tolerance Proportion Defective) represents the quality level routinely rejected by the sampling plan. The acceptance probability 0.10 at LTPD is defined as Consumer’s risk or beta risk. The AQL and LTPD are obtained using the Stationary OC curve for the periods of changing quality. Figure 3 shows the AQL and LTPD for the QSS-SS $n_r=20, a_r=0, s_r=1$ and $n_t=60, a_t=0, s_t=0$. The protection during the time of changing quality can be explained by using the maximum of Producer’s risk at AQL denoted by α_{max} and the maximum

of Consumer's risk at LTPD denoted by β_{\max} . The following equations are used for the calculating the maximum producer's and consumer's risk.

$$\alpha_{\max} = \max \begin{cases} \{1 - OC_r(AQL)\} \\ \{1 - OC_t(AQL)\} \end{cases} \quad (6)$$

$$\beta_{\max} = \max \begin{cases} \{1 - OC_r(LTPD)\} \\ \{1 - OC_t(LTPD)\} \end{cases} \quad (7)$$

The summary statistics include:

AQL	0.23%
LTPD	4.52%
α_{\max}	0.13
β_{\max}	0.415

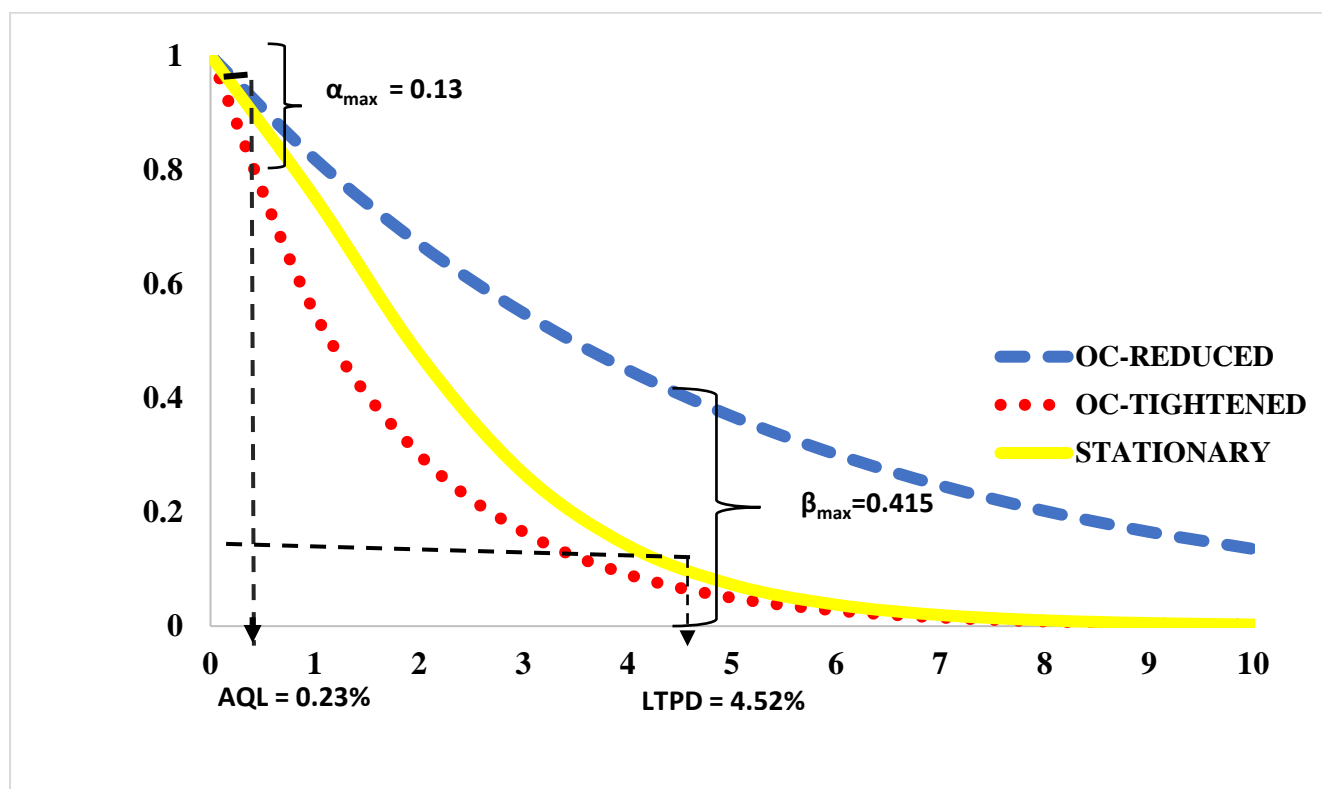


Figure 4: Summary Statistics

During the periods of changing quality, the producer's risk at AQL is normally 0.05, but can be as high as 0.13 and the consumer's risk at LTPD is normally 0.10, but can be as high as 0.415.

Increased Consumer Risk:

Figure 4 shows the transitive OC curves following a jump from zero percent defective. For the first lot following the jump, the consumer’s risk is 0.415. This is an increased risk of $0.415 - 0.1 = 0.315$. For the second lot, the consumer’s risk is 0.258. This is an increased risk of $0.258 - 0.1 = 0.158$. Moving from the first lot to second lot, the increased consumer’s risk is reduced by,

$$100 * \frac{(0.315 - 0.158)}{0.315} = 50 \%$$

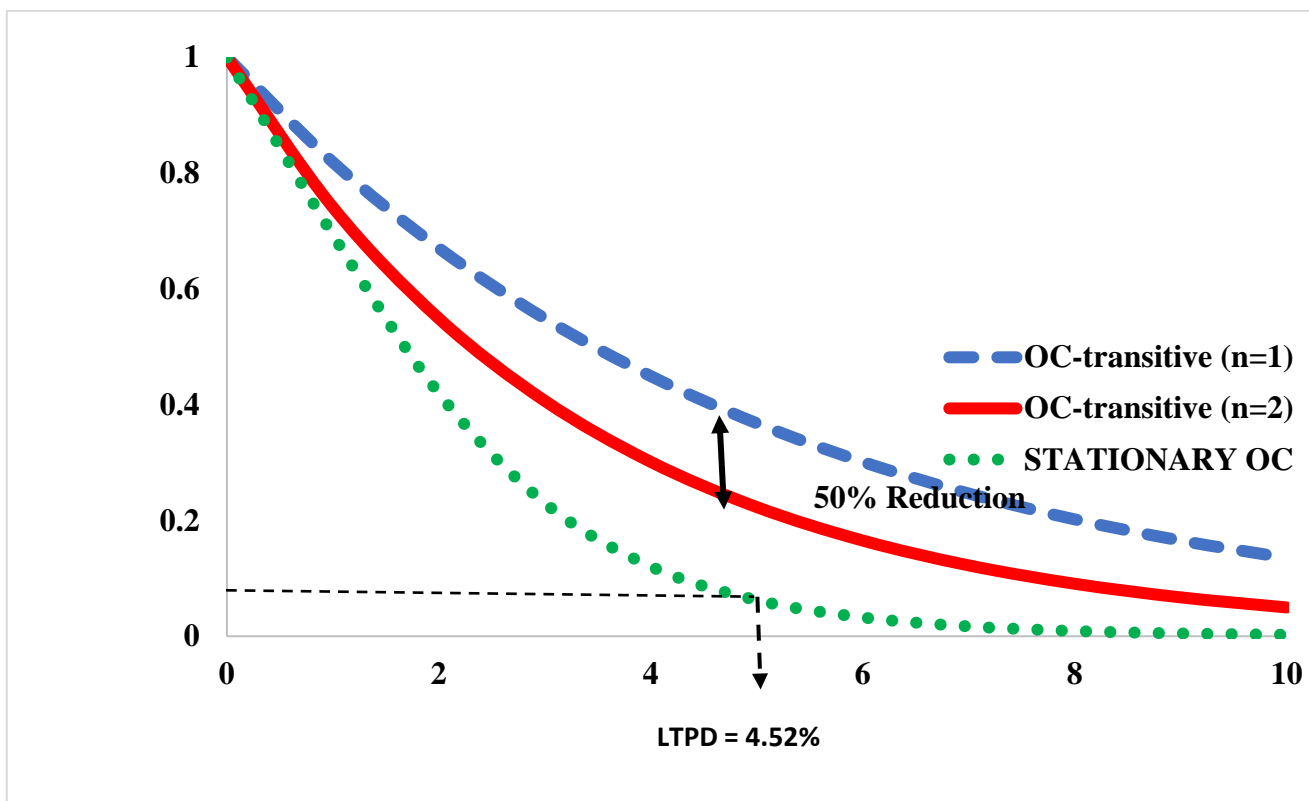


Figure 5 : Reduction of Increased Risk at LTPD

ASN and MTBS Curves:

The curve plotted against the average number of units inspected and the process percent defective is called Average Sample number (ASN) curve. The number of units inspected using QSS depends upon the lot quality inspected. Figure 5 displays the ASN curve of QSS-SS for the parameters $n_r=20$, $a_r=0$, $s_r=1$ and $n_t=60$, $a_t=0$, $s_t=0$ and for SSP ($n=80$, $a=1$). The ASN curve is close to $n_r=20$ for lower defective levels and reaches close to $n_t=60$ for higher defective levels. The ASN of a QSS-SS is given as follows.

$$ASN(p) = Pr_r(p) ASN_r(p) + Pr_t(p) ASN_t(p) \tag{8}$$

Where, $ASN_r(p) = n_r$ and $ASN_t(p) = n_t$ are the ASNs of the reduced and tightened plans respectively.

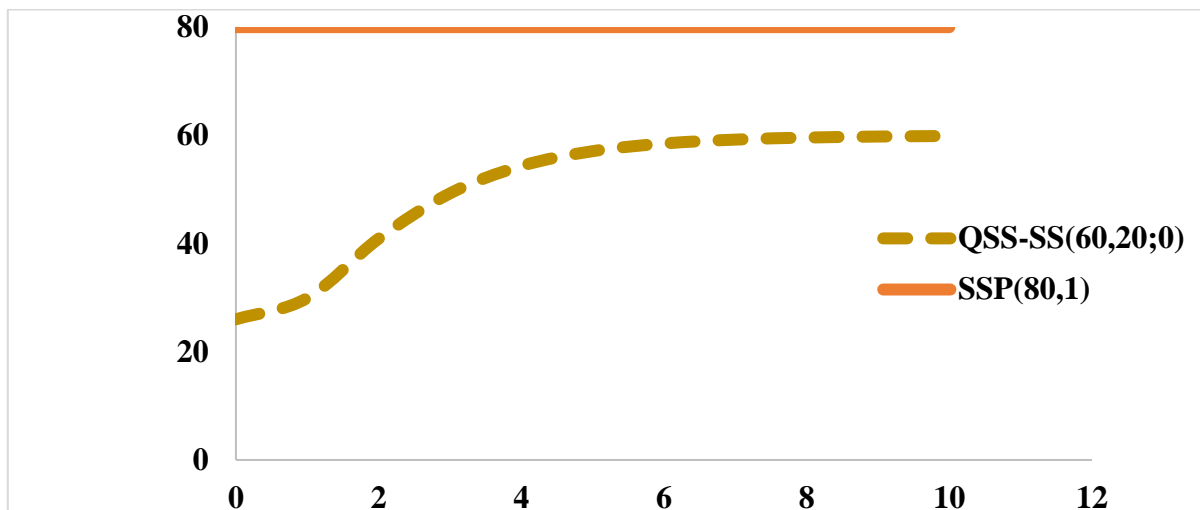


Figure 6: ASN Curve

MTBS:

Frequent switching of the sampling plan when the quality is unchanged can be considered as nuisance one. MTBS (Mean time Between Switches) is the average number of lots between switches and is calculated by,

$$MTBS_{(p)} = \frac{\frac{1}{S_{r \rightarrow t(p)}} + \frac{1}{S_{t \rightarrow r(p)}}}{2} \tag{9}$$

$S_{r \rightarrow t(p)}$ be the switching probability of reduced plan and $S_{t \rightarrow r(p)}$ be the switching probability of the tightened plan. Therefore, the expected number of lots inspected before switching is $\frac{1}{S_{r \rightarrow t(p)}}$. Similarly, $\frac{1}{S_{t \rightarrow r(p)}}$ be the expected number of lots inspected before switching back to reduced. The addition of two quantities divided by two since two switches are involved. Figure 6 shows the MTBS curve plotted for the mean time between switches and process average. QSS -SS are found to be constant at low and high defective rates but switch frequently at the intermediate values.

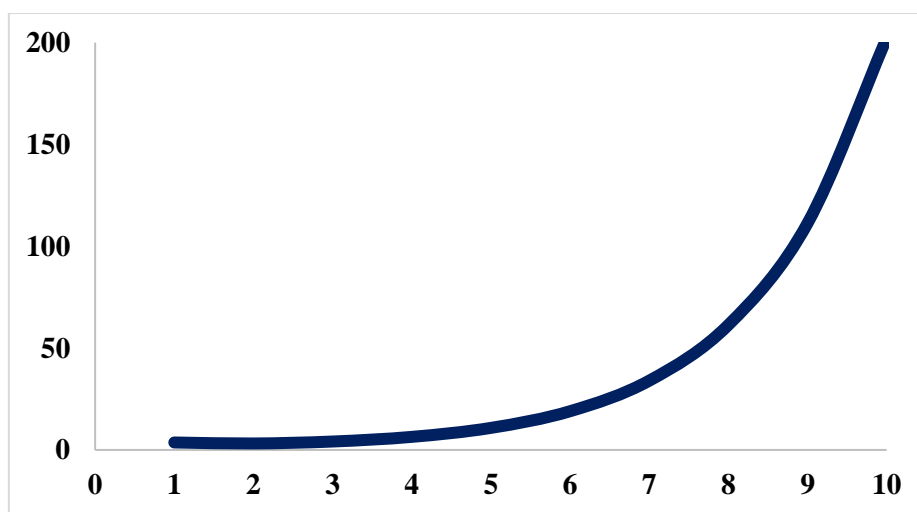


Figure 7: Plot MTBS against Process Average Defective

Table 1 shows the comparison of ASN for AQL=1% AND LTPD = 4% at 1% defective. The comparison is made between SSP, DSP, QSS and QSS-SS.

Table 1: Different Sampling Plans with AQL=1% and LTPD = 4% at 1% defective

TYPE	PARAMETERS	ASN
SINGLE	$n=198, a=4$	198
DOUBLE	$n_1 = 105, a_1 = 1, r_1 = 4, n_2 = 156, a_2 = 5$	146
QSS (ROMBOSKI)	$n_n=136, a_n=3, n_t=167, a_t=3$	137
QSS-SS(TAYLOR)	$n_r = 119, a_r=3, n_t=167, a_t=3, s_t=1$	120

Conclusion:

QSS-SS can be utilized to improve protection during the time of changing quality (transitive OC curves). The increased consumer risk and producer risk and the reduction in consumer's risk moving from the first lot to second are calculated by using the Stationary OC curves. This QSS -SS offer economic alternatives to Single Sampling Plan, Double Sampling Plan, Quick Switching System (Normal to Tightened) and the probability of acceptance is found to be higher than other plans and QSS. They can be used to significantly minimize the number of units during the inspection process and simpler to use.

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