

Unsteady two Dimetional Stretching Flow Of a Viscous Incompressible Fluid in The Presence Of a Naturally Permeable Boundary

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Abstract

The present study considers the unsteady two-dimensional flow of a viscous fluid through a channel bounded by a flat deformable sheet and a naturally permeable lower boundary. Results from two-dimensional simulations are documented for discussion on velocity distributions, stream functions, pressure coefficients, and skin friction coefficients along the stretching sheet and the porous boundary. All equations have been solved and the results have been graphically presented using MATLAB.

Keywords: Viscous fluid. Deformable Sheet, Permeability, Porous medium, etc.

Introduction

Because of the numerous scientific and engineering applications, the study of the fluid flow in the presence of naturally permeable boundaries is very much justified. Beavers and Joseph⁽¹⁾ suggested an interface slip condition between fluid and porous medium. Saffran⁽²⁾ theoretically justified the slip condition of Beavers and Joseph and further modified it for the case when the permeability of the porous medium is very small so as to calculate the outer free fluid flow near the porous boundary. This condition is another one that has been widely used in many other problems of fluid flow in porous media. Flow stability and uniqueness have been considered by Bhattacharyya and Gupta⁽³⁾, Mcleod and Rajagopal⁽⁴⁾ due to a stretching boundary. Chauhan⁽⁵⁾ studies slip effects on a non-Newtonian fluid through a porous medium channel with a stretching wall. Xu Hang et al⁽⁶⁾ created a mathematical model for flow in a micro channel driven by an upper stretching wall.

This is the effect of the tendency of a viscous fluid to flow through a channel capped by a flat elastic deformable sheet and naturally permeable beds of very small permeability when this sheet is stretched in its own plane pointing an outward flow velocity proportional to distance from a particular point in it. In the absence of any other external pressure gradient and small permeability of the porous medium, the flow inside the porous medium can be constituted as zero, and the effect of the permeability trap in the outer flow inside that channel comes through Saffran's⁽⁷⁾simplification to the Beavers and Joseph slip condition. The expressions for velocity, stream flow function, pressure coefficient, and factors of skin friction on the stretching sheet and the porous boundary were derived and considered. Current Shekhawat⁽⁸⁾ studies A Two-Dimensional Stretching flow of a Viscus Incompressible Liquid in the Presence of a Naturally Permeable Boundary. Now Singh &Singh⁽⁹⁾study The Unsteady Two-Dimensional Stretching flow of a Viscus Incompressible Fluid in The Presence of a Naturally Permeable Boundary.

USE SYMBOL

K	Thermal Conductivity.
α	is a non-dimensional constant depending upon the porous material
p	the pressure in the channel
u and v	are the velocity components in the channel.

h	With a channel through fluid
R	small values of the stretching parameter
ϑ	Kinematic Viscosity of the Fluid
p	Density of the Fluid
A	Constant of the Fluid

FORMULATION

The Unsteady two-dimensional steady laminar flow of a viscous incompressible fluid through a channel of width h is considered. The channel is bounded by a naturally permeable bed and a stretching sheet. Let Suppose $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

The flow in the channel (free fluid region, $(0 \leq y \leq h)$) is governed by the following Navier-Stokes equations:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \frac{-1}{p} \frac{\partial p}{\partial x} + \vartheta \frac{\partial^2 u}{\partial x^2} \quad (1)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} = \frac{-1}{p} \frac{\partial p}{\partial x} + \vartheta \frac{\partial^2 v}{\partial x^2} \quad (2)$$

The flow through porous media is governed by Darcy's law. However, one assumes zero filter velocity of the porous medium when there is an absence of any external pressure gradient, with small permeability interior flows contributing little to the exterior free fluid flow. Under this assumption, the boundary conditions in the channel flow are given as follows

$$Y = 0 \text{ at } u = \frac{k}{\alpha} \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

$v=0$, and continuity of pressure,

and at $y=h, u=cx$

SOLUTION OF MATHEMATICAL EQUATION -

We solved this equation no (1) and (2) by using MATLAB programming and making graphs with help of p of MATLAB codes. By equation (1)

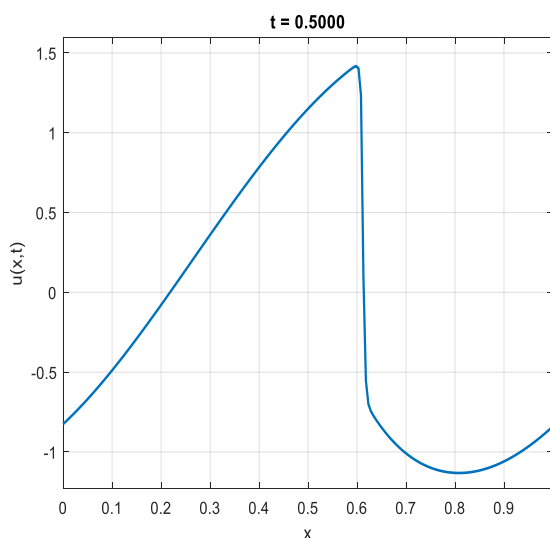


Fig. 1

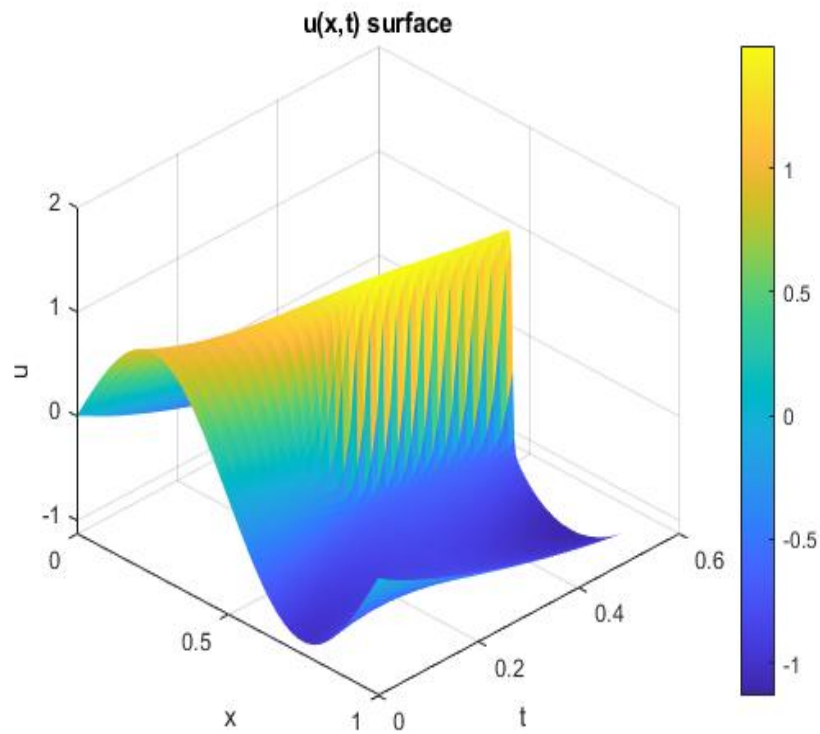


Fig. 2

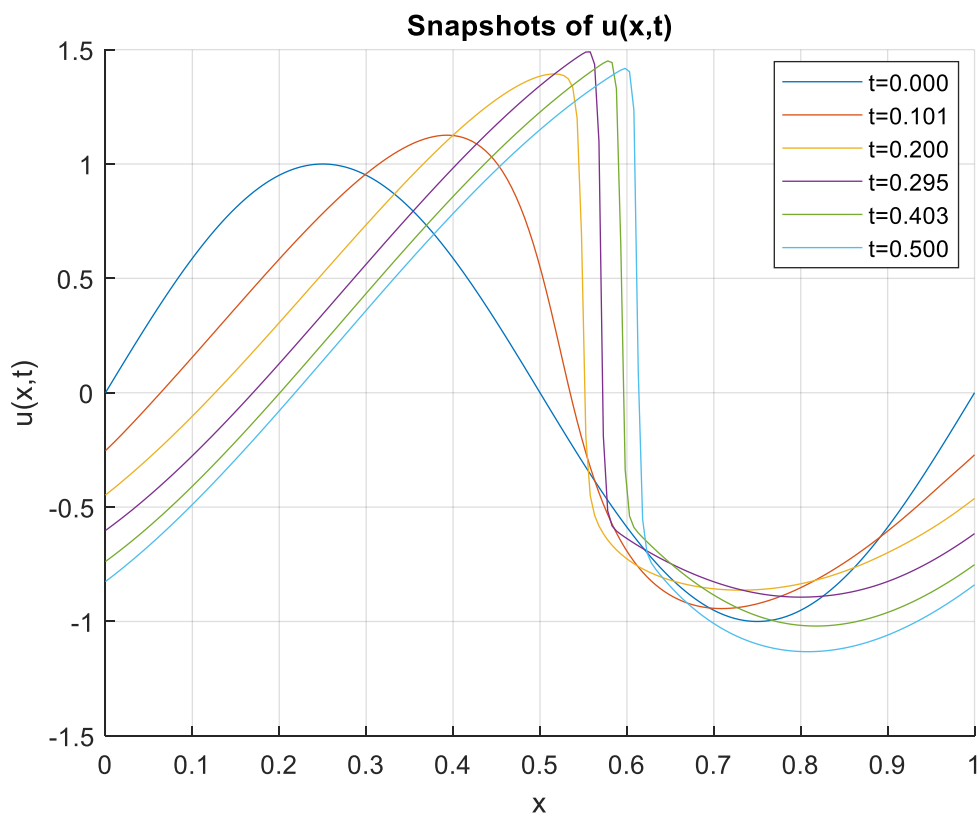


Fig. 3

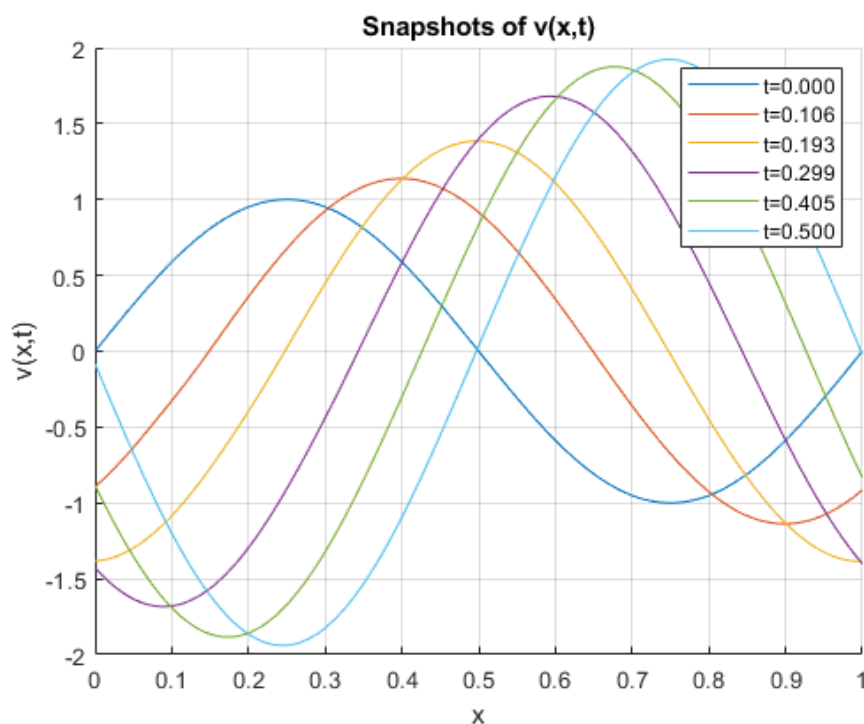


Fig. 4

According to equation (2)

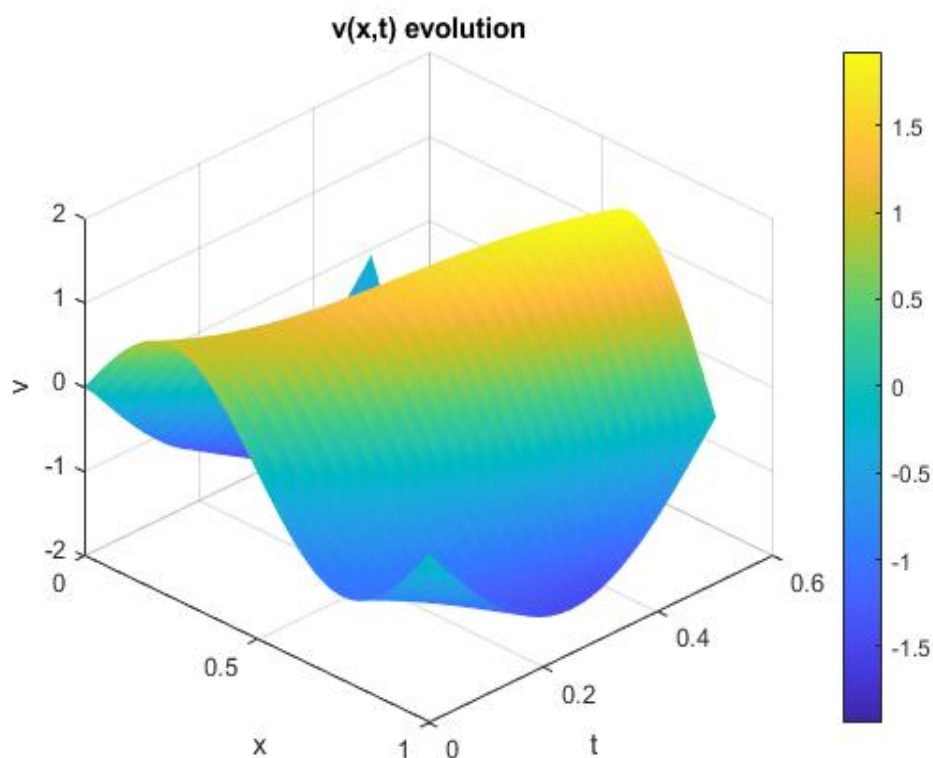


Fig. 5

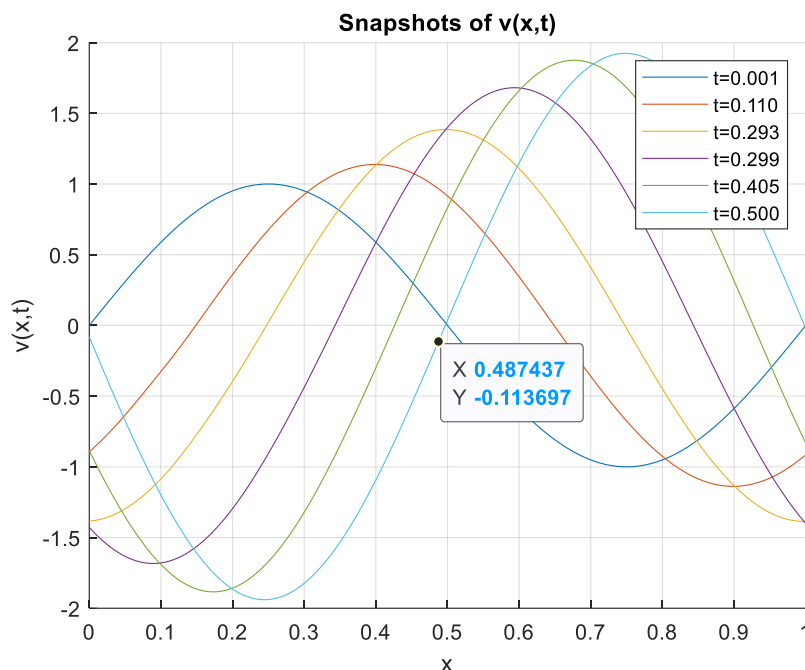


Fig. 5

DISCUSSION-

The study of unsteady two-dimensional stretching flow of a viscous incompressible fluid with a naturally permeable boundary highlights the combined effects of time-dependency, stretching motion, and boundary permeability on the flow and heat transfer characteristics. The governing nonlinear differential equations were solved using similarity transformations and suitable numerical/analytical methods, enabling a detailed investigation of the velocity and boundary layer profiles. The flow behavior in the presence of a naturally permeable boundary has been examined in a channel bounded by a deformable sheet. The stretching of it induces a backward flow in the channel near the permeable lower boundary. The temperature profiles are drawn in figures 1 and 2. Surface effect by temperature is given in 3 figures.

CONCLUSION -

The analysis of unsteady two-dimensional stretching flow in the presence of a naturally permeable boundary provides valuable insights into the dynamics of incompressible viscous fluids. The study concludes that unsteadiness enhances velocity gradients and reduces boundary layer thickness. Suction through the permeable boundary improves flow stability and wall shear, while injection destabilizes the boundary layer. The permeability parameter serves as an effective control mechanism for boundary layer thickness and drag forces. The combined effects of stretching, unsteadiness, and permeability yield complex flow patterns that are crucial for optimizing industrial and natural processes.

Reference –

- (1)Skiadas, B. C., "Boundary-Layer Behaviour on Continuous Solid Surfaces," *AICHE Journal*, 1961 foundational work on boundary layers over moving continuous surfaces.
- (2) Crane, L. J., "Flow Past a Stretching Plate," *Journal of (appl.) Mechanics / engineering literature*, 1970 — classical similarity solution for flow due to a stretching sheet.

- (3) Zheng, L., Wang, L., & Zhang, X., "Analytic Solutions of Unsteady Boundary Flow and Heat Transfer on a Permeable Stretching Sheet with Non-Uniform Heat Source/Sink," *Communications in Nonlinear Science and Numerical Simulation*, 2011.
- (4) Bachok, N., "Unsteady three-dimensional boundary layer flow due to a permeable shrinking sheet," *Applied Mathematical Modelling / Journal (2010)* — similarity solutions for unsteady permeable shrinking/stretching surfaces. [SpringerLink](#)
- (5) Roşca, N. C., "Unsteady boundary layer flow over a permeable curved stretching/shrinking surface," *International Journal / journal article (2015)* — curved surface, unsteady, mass-suction (permeable) effects. [ScienceDirect](#)
- (6) Choudhary, M. K., "Unsteady MHD Flow and Heat Transfer over a Stretching Sheet," *Science Direct article (2015)* — two-dimensional unsteady viscous electrically conducting fluid over a stretching sheet (porous / permeable extensions appear). [ScienceDirect](#)
- (7) Hamad, M. A. A., "Similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet," *Applied Mathematics and Mechanics (English)*, 2012 — numerical similarity solutions and unsteady parameter studies. [amm.shu.edu.cn](#)
- (8) Cai, W., "Unsteady Convection Flow and Heat Transfer over a Vertical Stretching Surface," *PMC / 2014* — unsteady convection over a permeable stretching surface (heat transfer emphasis). [PMC](#)
- (9) Hunegnaw, H. D., "Unsteady Boundary Layer Flow of Williamson Nanofluids over a Heated Permeable Stretching Sheet," *Research PDF / 2021* — numerical study for unsteady permeable stretching sheet with viscous dissipation / porous medium. [Semantic Scholar](#)
- (10) Vanitha, G. P., "Casson nano liquid film flow over an unsteady moving surface," *PMC / 2023* — unsteady film flows over moving/permeable surfaces (useful for modern nanofluid extensions). [PMC](#)
- (11) Weidman, P. D., "Generalized Crane flow induced by continuous surfaces (permeable sheet generalization)," *Springer / 2010* — generalization of Crane's solution including transpiration/permeation at the wall.
- (12) Beavers, G. S., Joseph, D. D., Boundary conditions at a naturally permeable wall. *Journal of fluidmechanics*, 30(1) (1967) 197-207.
- (13) Saffman P. G., On the boundary condition at the surface of a porous medium. *Studies in appliedmathematics*, 50(2) (1971) 93-101.
- (14) Rudraiah, N.Chandrasekhara, B. C.Veerabhadraiah, R., & Nagaraj, S. T.Some flow problems in porousmedia, *PGSAM*, Ser 2 (1979) 13-16.
- (15) BhattB.S.SachetiN.C. Effect of rotation on Rayleigh layers in presence of naturally permeableboundaries. *Journal of engineering mechanics*, 113(11) (1987) 1795-1800.
- (16) Kuznetsov, A.V., Analytical investigation of fluid flow in the interface region between a porous medium and a clear fluid in channels partially filled with a porous medium, *Appl. Sci. Res.* 56 (1996) 53-67.
- (17) Kuznetsov, A.V., Analytical investigation of Couette flow in a composite channel partially filled with a porous medium and partially with a clear fluid., *Int. J. of Heat and Mass Transfer*, 41 (1998) 2556-2560.
- (18) Bhattacharyya, S. N., Gupta, A. S., On the stability of viscous flow over a stretching sheet. *Quarterly of applied mathematics*, 43(3) (1985) 359-367.
- (19) McLeod, J. B., Rajagopal, K. R., On the uniqueness of flow of a Navier-Stokes fluid due to a stretchingboundary, *Archive for Rational Mechanics and Analysis*, 98 (1987) 385-393.

(20) Chauhan, D.S. and Aggarwal, and A. Olkha, Slip effect on non-Newtonian fluid flow through porous medium in a channel of lower stretching wall: Homotopy Analysis Method, Acta Technica, 58 (2013) 173 - 187.